

# FUNDAMENTOS DE CRISTALIZACIÓN

Juan Manuel García-Ruiz  
CSIC-Universidad de Granada

Insubria International Summer School  
Crystallography for Health and Biosciences



La cristalización es un fenómeno que consiste en dos procesos diferentes. El primero de ellos se llama **nucleación**, y consiste en la formación de clusters nanoscópicos de la nueva fase sólida que pueden crecer hasta formar cristales macroscópicos de forma irreversible. El segundo es el proceso de **crecimiento del cristal**.

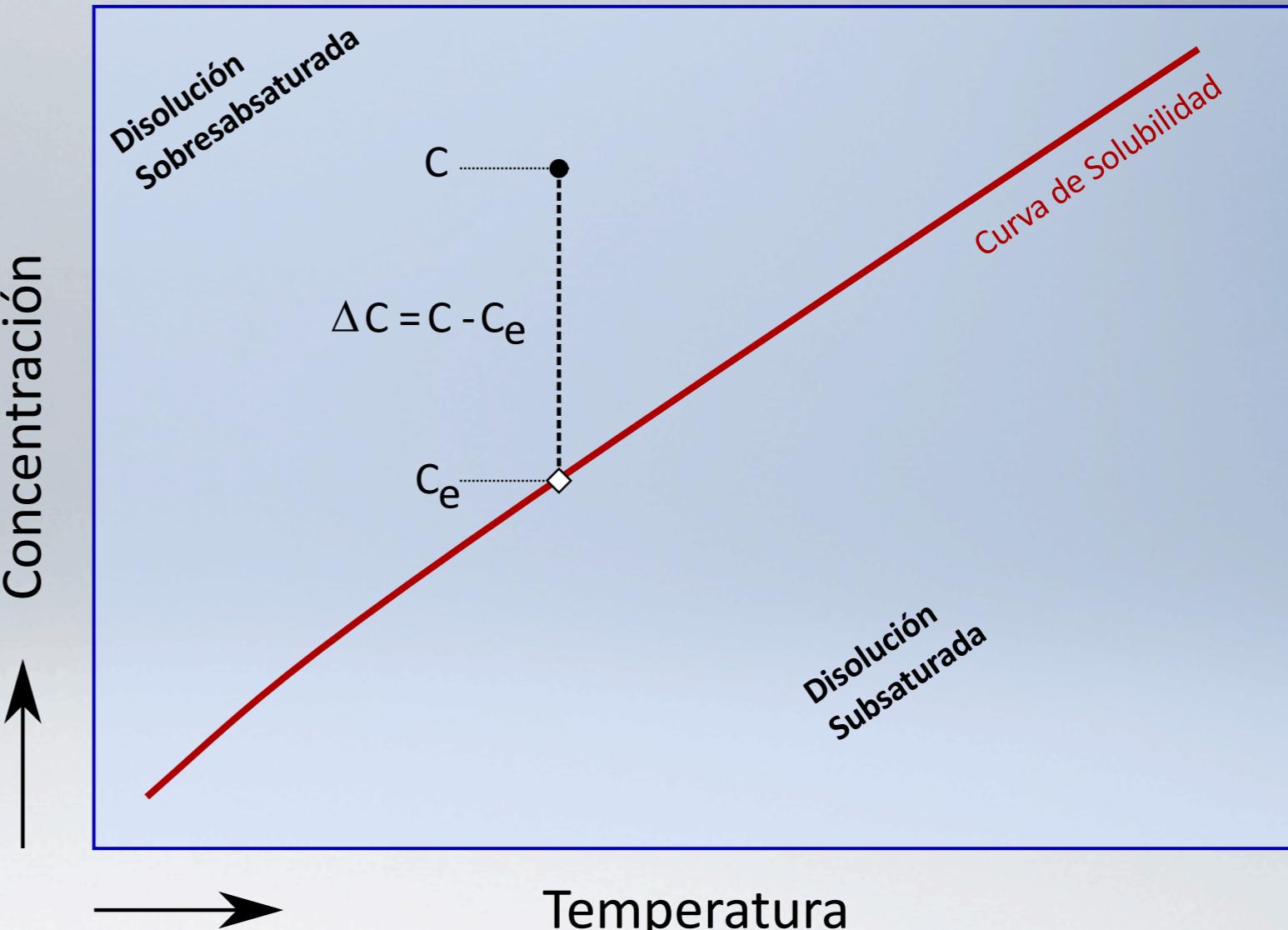
# Recordando conceptos sencillos pero importantes

Las disoluciones subsaturadas  
son termodinámicamente  
estables

Las disoluciones saturadas  
están en equilibrio

Las disoluciones  
sobresaturadas son  
termodinámicamente inestables

Una solución sobresaturada  
debería crear una fase sólida.  
Sin embargo, esto sólo es  
cierto cuando la  
sobresaturación alcanza un  
valor crítico.



# Recordando conceptos sencillos pero importantes

**Disolución:** fase homogénea que contiene moléculas de soluto y las moléculas de disolvente

**Concentración de equilibrio o solubilidad ( $C_e$ ):**

La concentración de soluto en un disolvente en equilibrio con soluto no disuelto, a una temperatura y presión dadas.

**Disolución saturada:**

**Disolución sobresaturada:**

**Disolución subsaturada:**

$$C = C_e$$

$$C > C_e$$

$$C < C_e$$

**Expresiones de la sobresaturación**

**Sobresaturación :**

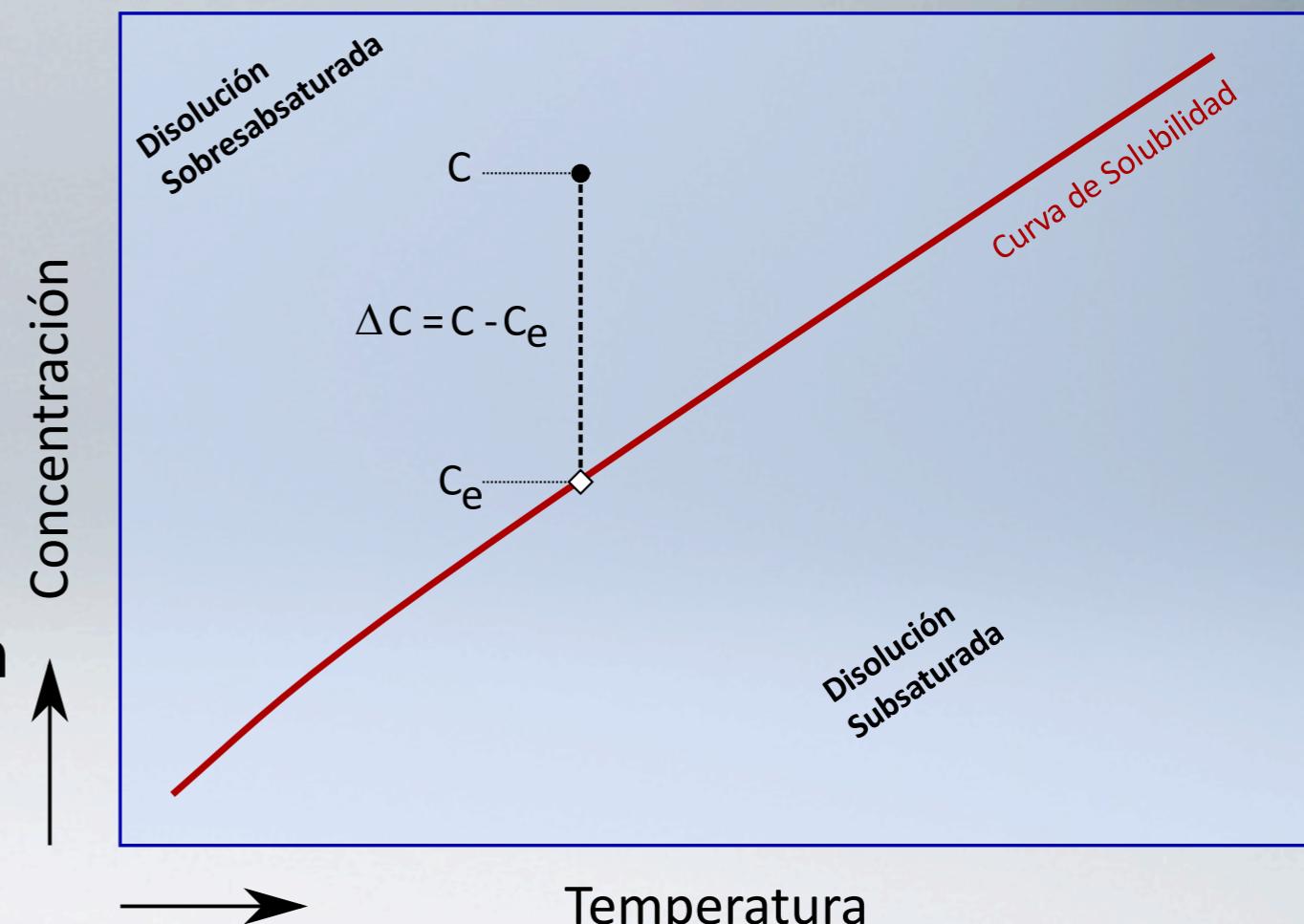
$$C - C_e$$

**Razón de sobresaturación:**

$$\frac{C}{C_e}$$

**Sobresaturaciónn relativa:**

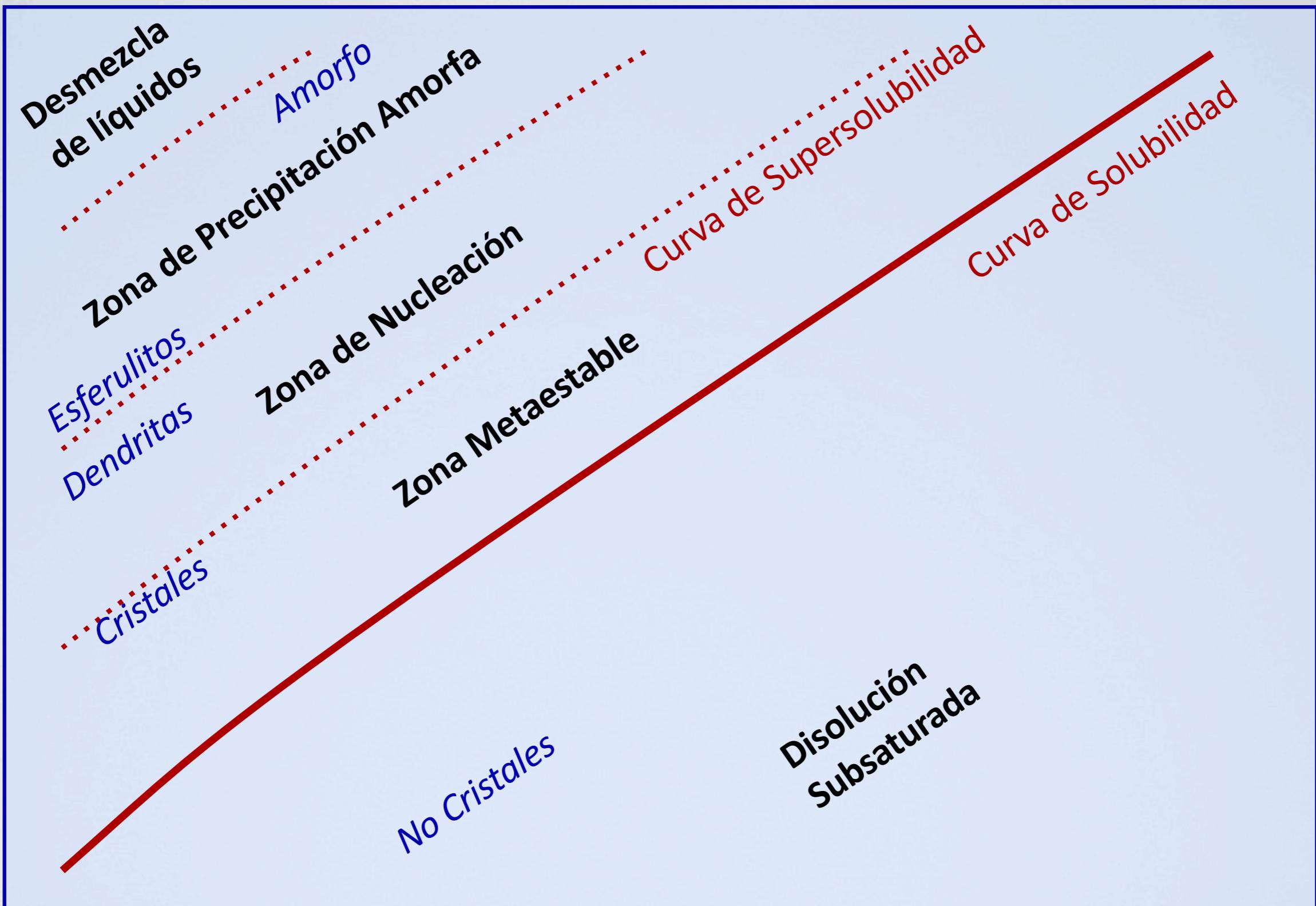
$$\frac{C - C_e}{C_e}$$



Concentración



Temperatura



¿Por qué hay una **zona metaestable** donde los cristales pueden crecer pero no pueden nuclear, es decir, no pueden formarse

Una visión intuitiva del proceso de nucleación

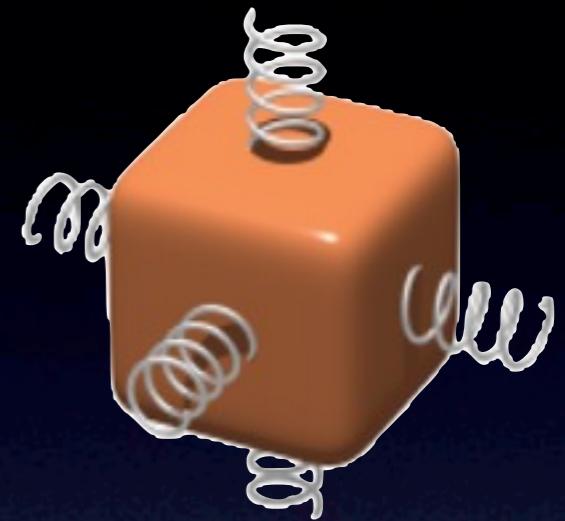
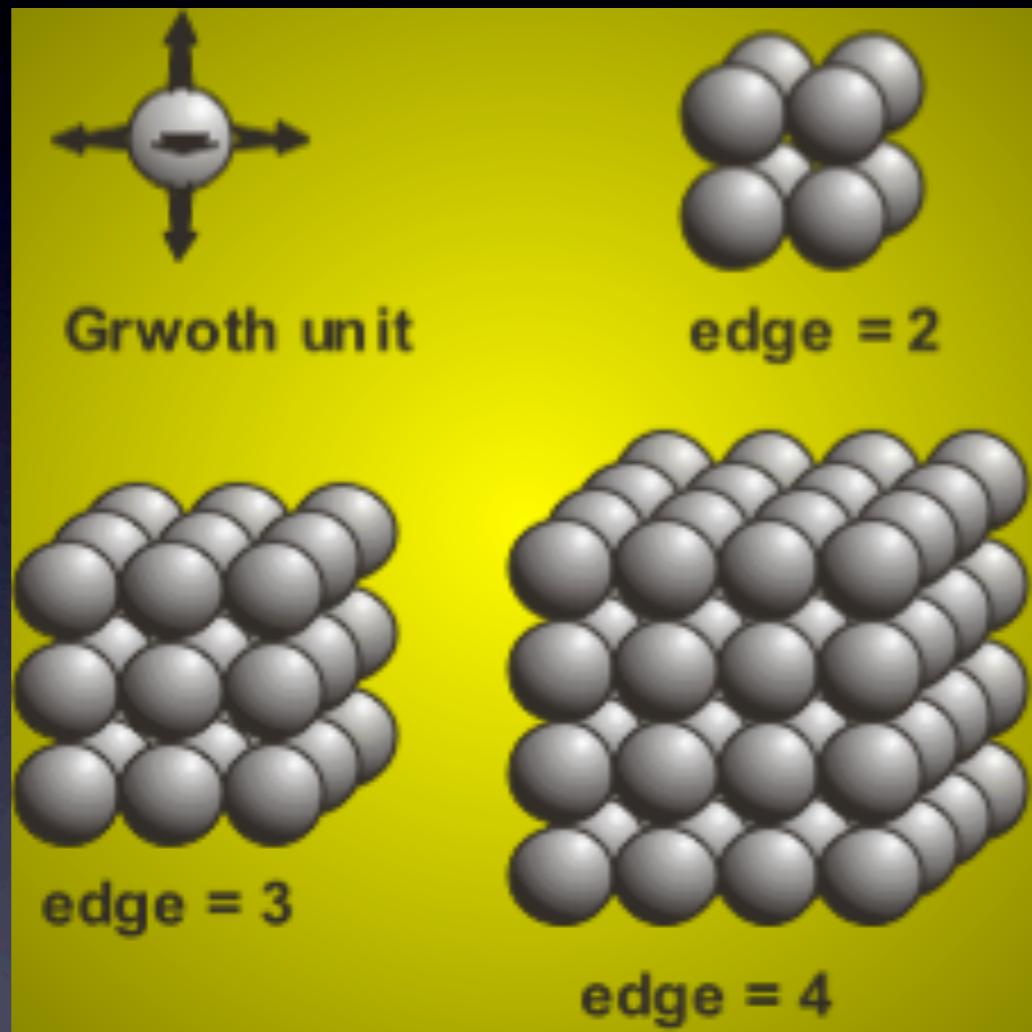
# Nucleaciónn

Simulación de nucleación

When a cluster form, a **surface** is created that separates two **volumes** (the cluster and the mother solution) quite different from a structural viewpoint



**El cristal Kossel.** La unidad virtual de crecimiento tiene seis enlaces insaturados situados perpendicular a cada cara del cubo

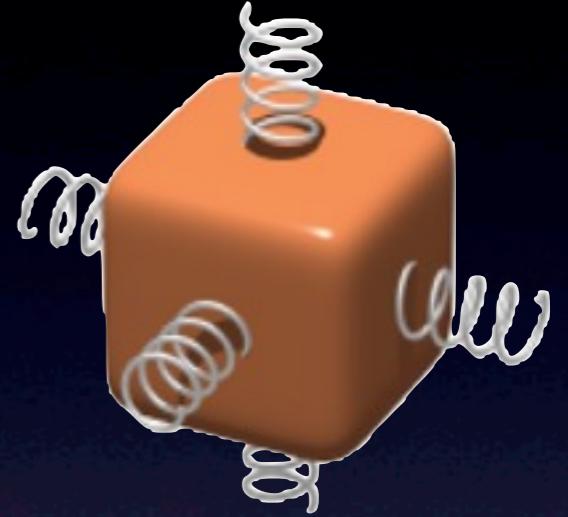


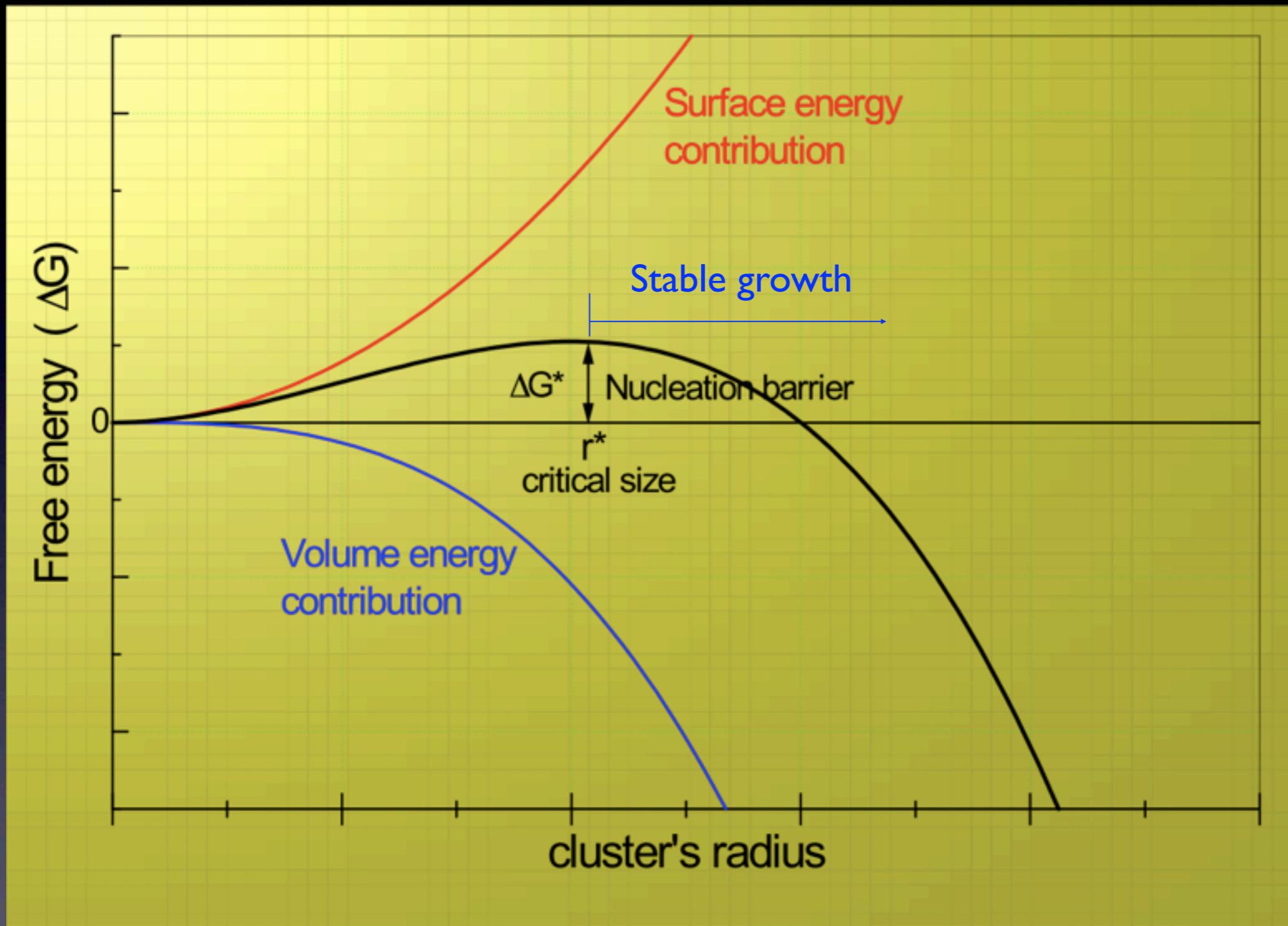
La cohesión del cluster es proporcional al número de enlaces saturados internos versus el número de enlaces en dirección hacia la solución madre

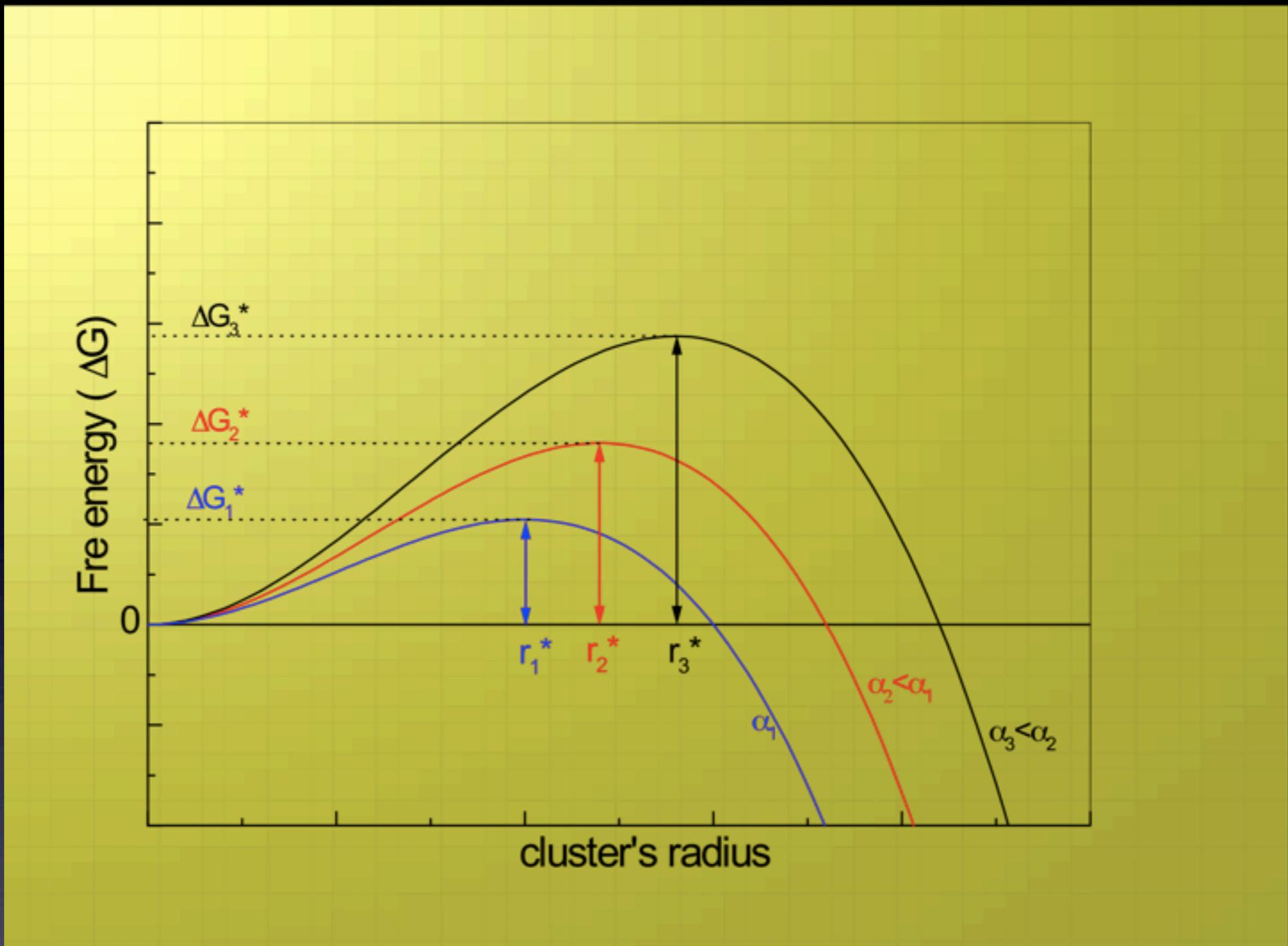
J. M. García-Ruiz, Nucleation of protein crystals, *Journal of Structural Biology*, 142 (2003) 22-31.

Cohesion energy is proportional to the cluster's volume  
 The probability to disaggregate is proportional to the cluster's surface

$a$	$F_S = 6 * a^2$	$F_A = a^3$
1	$6 \times 1^2$	$a (1)^3$
2	$6 \times 2^2$	$a (2)^3$
3	$6 \times 3^2$	$a (3)^3$
4	$6 \times 4^2$	$a (4)^3$
5	$6 \times 5^2$	$a (5)^3$
6	$6 \times 6^2$	$a (6)^3$
7	$6 \times 7^2$	$a (7)^3$
8	$6 \times 8^2$	$a (8)^3$
9	$6 \times 9^2$	$a (9)^3$



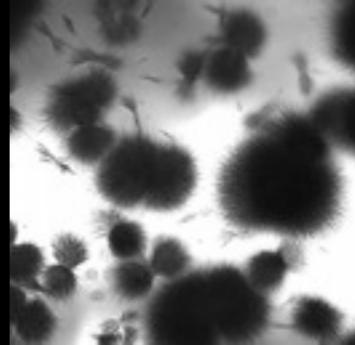
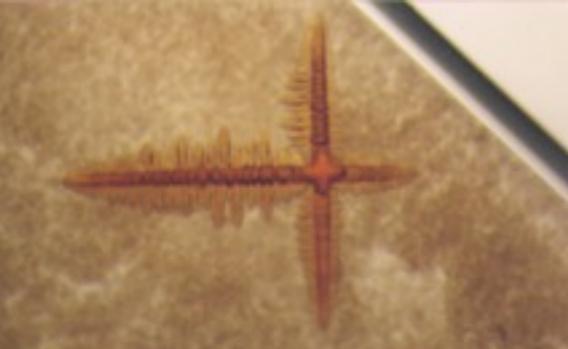
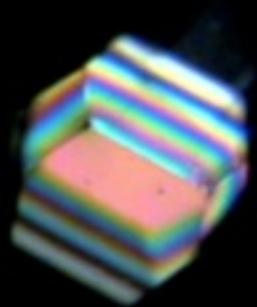
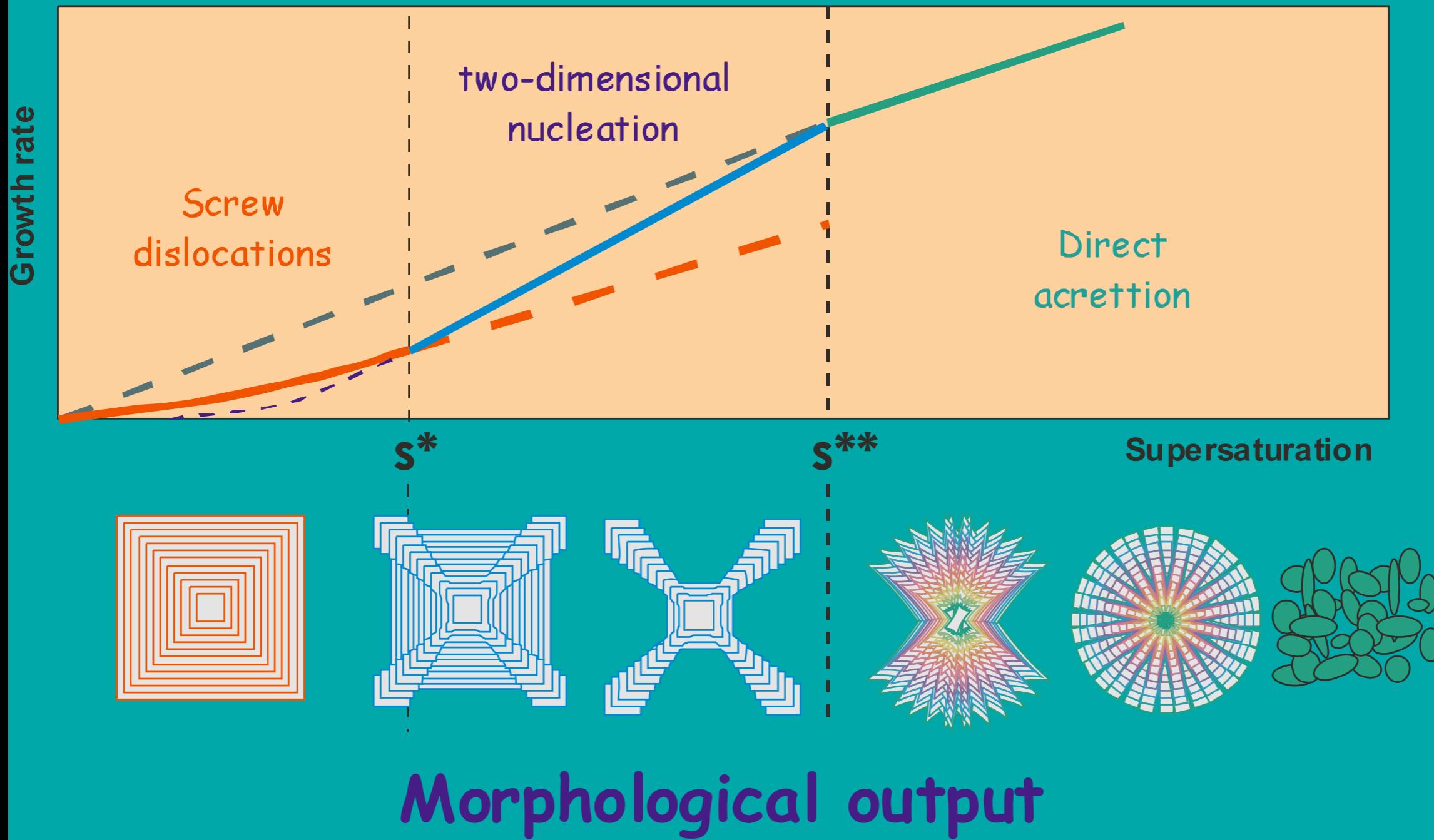




$$r^* = \frac{2\nu\gamma}{kT \ln S}$$

$$\Delta G^* = \frac{16\pi\nu^2\gamma^3}{3[kT \ln S]^2}$$

# Growth mechanisms



According to the Boltzman's law, the probability of a fluctuation of magnitude  $W$  is given by :  $e^{-\frac{W}{kT}}$ . Thus, the probability of a nucleus of size  $i+1$  growth units being created from a cluster of size  $i$  is:  $e^{\frac{-\Delta G_i}{kT}}$ , where  $\Delta G_i$  is the change of free energy associated to the addition of one growth unit to a cluster of size  $i$ . As  $n\Delta G_i = \Delta G$ , then,

$$\frac{N_n}{N_1} = \exp\left(\frac{-\Delta G}{kT}\right)$$

The nucleation frequency  $J$ , i.e. The number of nuclei per unit of volume and unit of time that achieve the critical size can be expressed as:

$$J = \kappa_0 \exp\left(\frac{-\Delta G^*}{kT}\right)$$

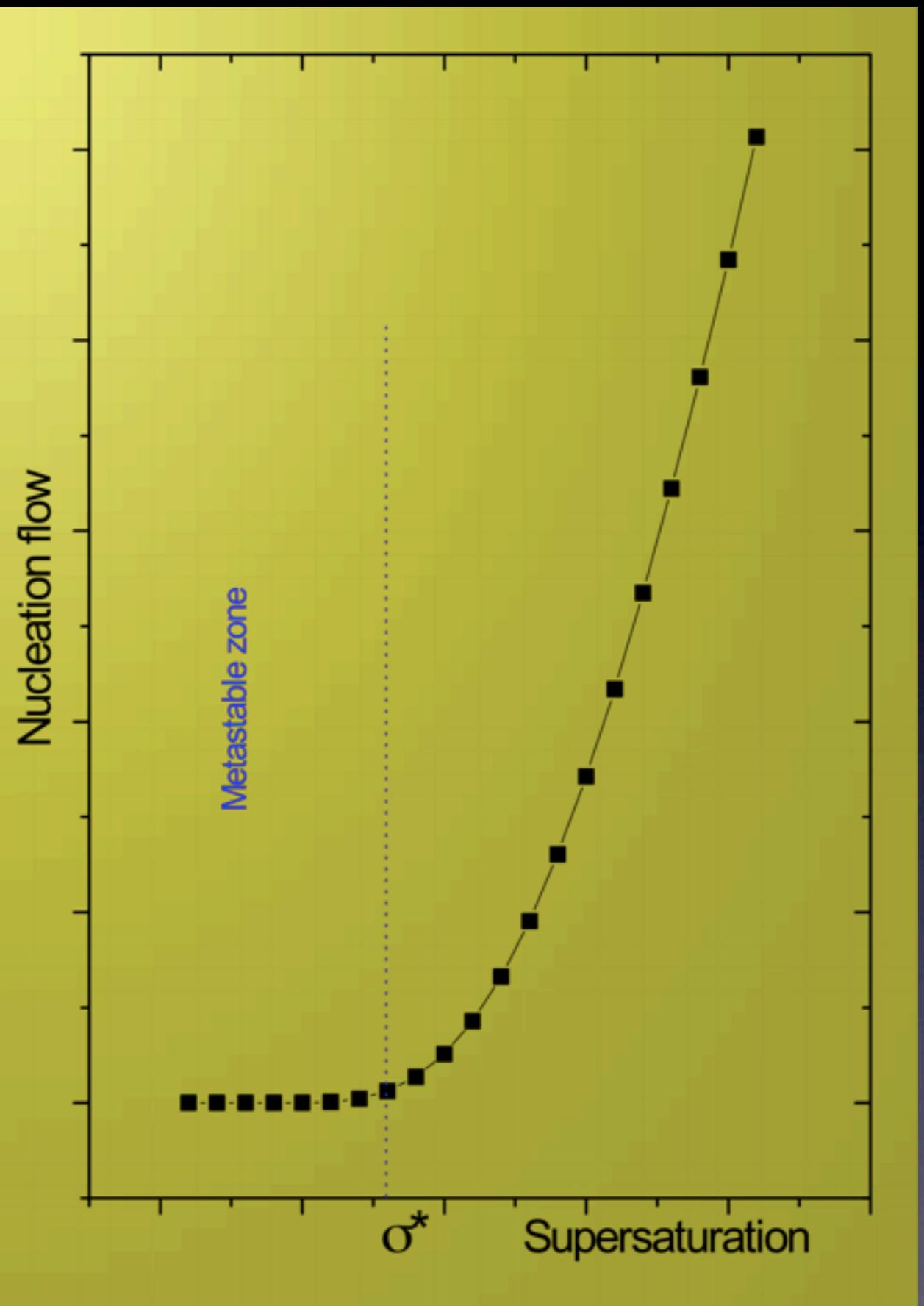
Substituting the value of  $\Delta G^*$  in the last equation we got:

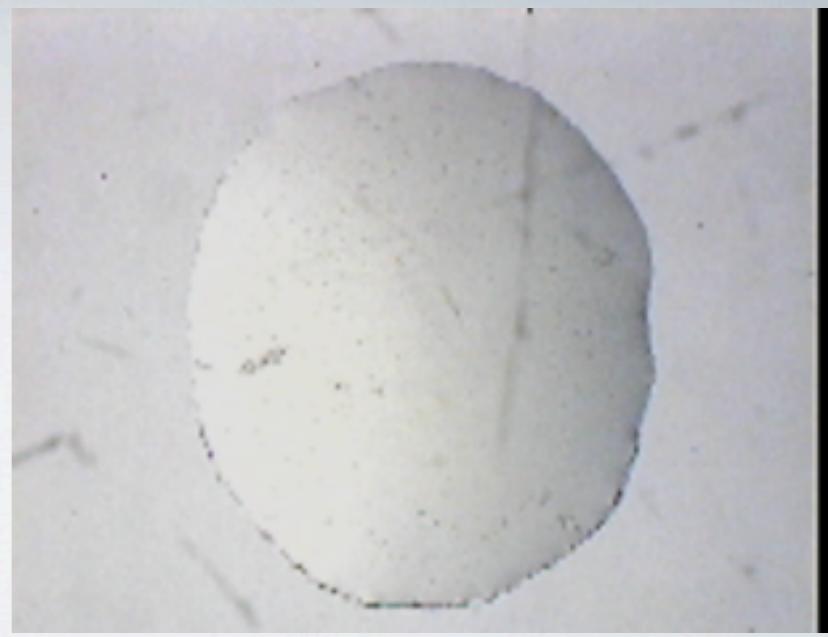
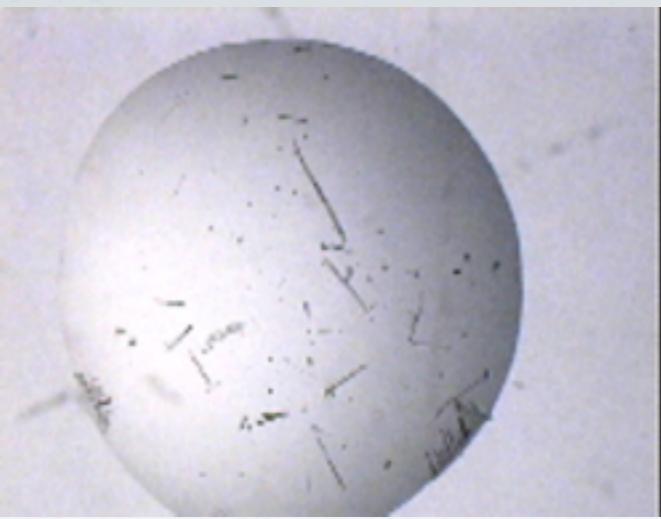
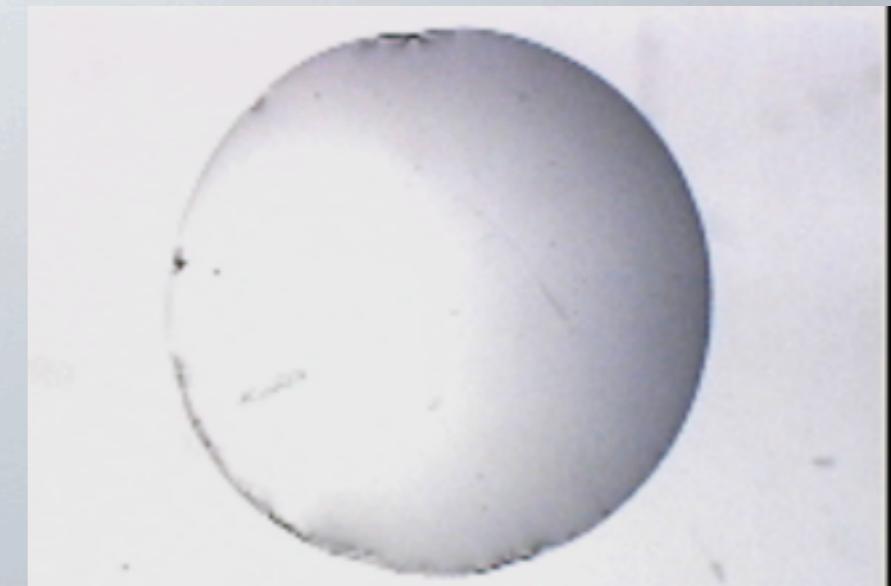
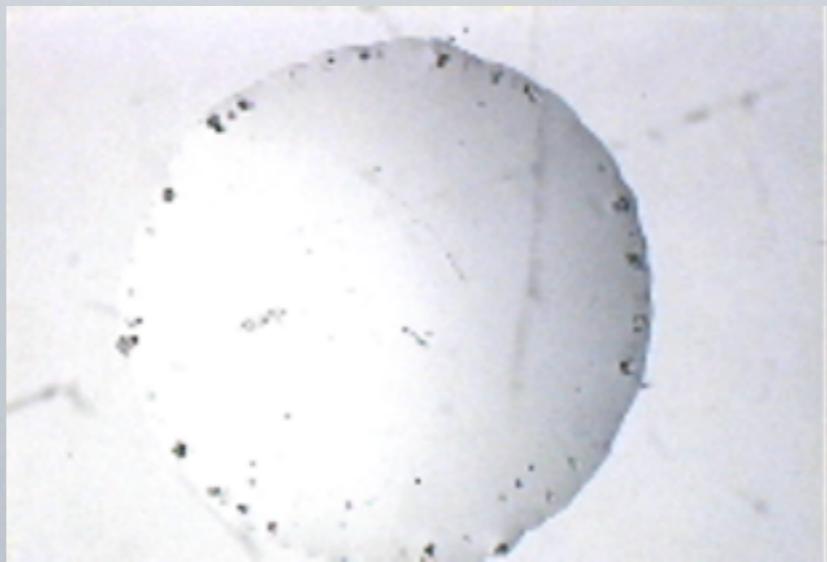
$$J = \kappa_0 \exp\left(-\frac{16\pi v^2 \gamma^3}{3(kT)^3 [\ln S]^2}\right)$$

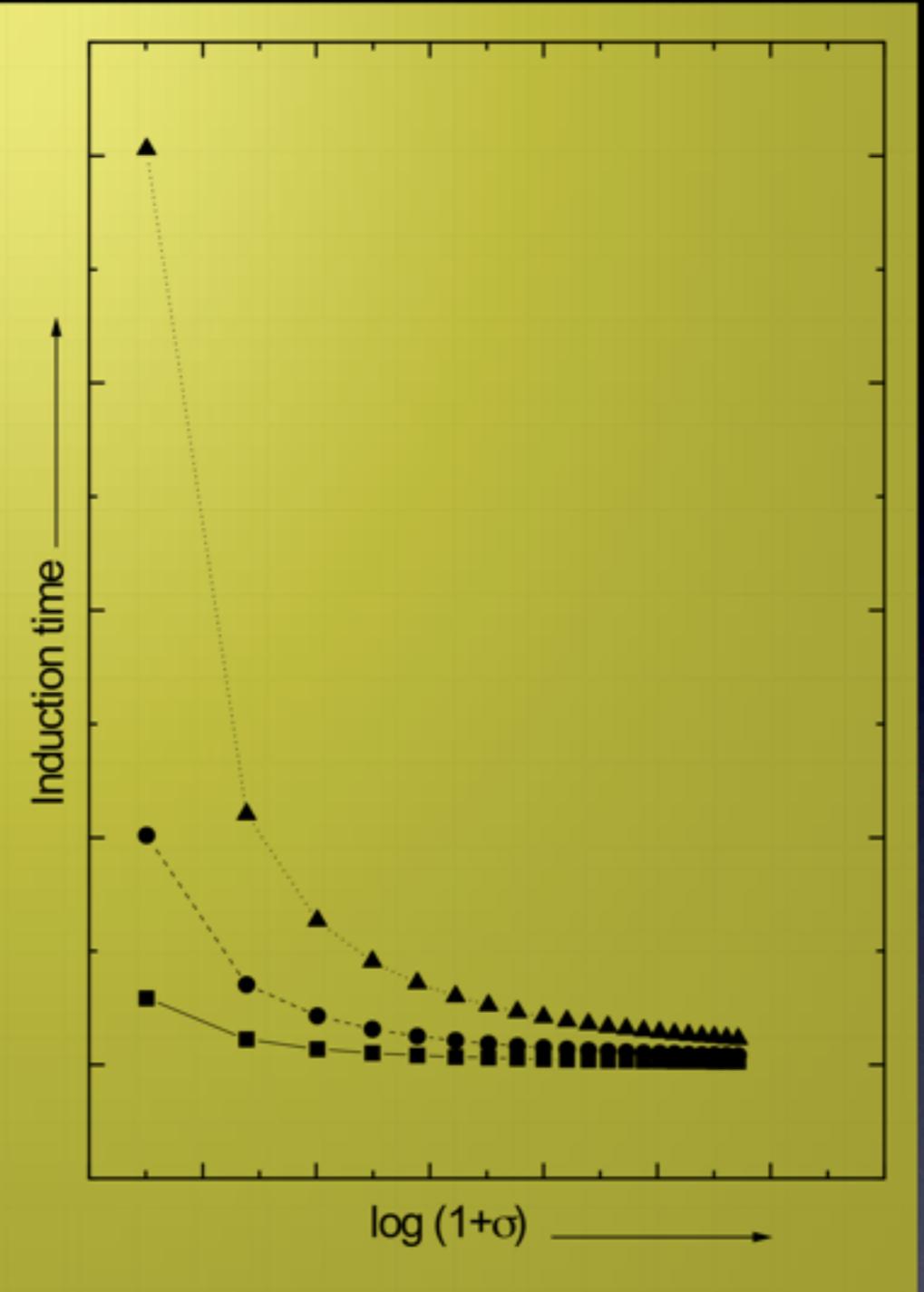
**Nucleation frequency  $J$  is defined as the number of stable nuclei forming per unit of time and unit of volumen.**

**It is given by:**

$$J = \kappa_0 \exp\left(-\frac{16\pi v^2 \gamma^3}{3(kT)^3 [\ln S]^2}\right)$$







$$t_N = \frac{\kappa^{te}}{J_N} = \kappa_1 \exp\left(\frac{\kappa_2}{T^3 \ln(1 + \sigma)^2}\right)$$

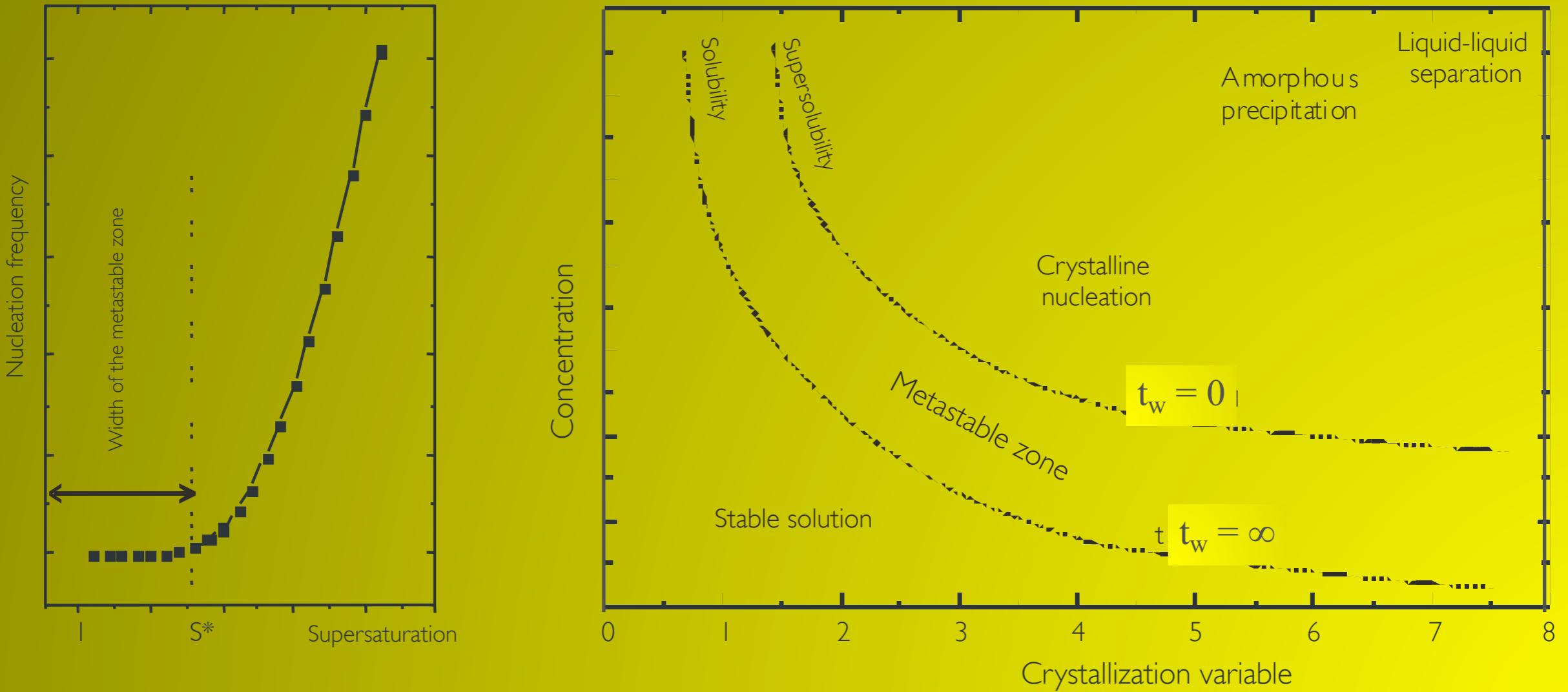
The induction time is the reverse of the nucleation flow and is the summation of three factors:

$$t_n = t_d + t_{n^*} + t_g$$

The first is the time required to get a stationary distribution of size of precritical size, the second, the waiting time to obtain clusters of a critical size and the third one, is the waiting time for a stable cluster to reach the size to be detectable. This last parameter depends strongly of the experimental technique used to detect nucleation.

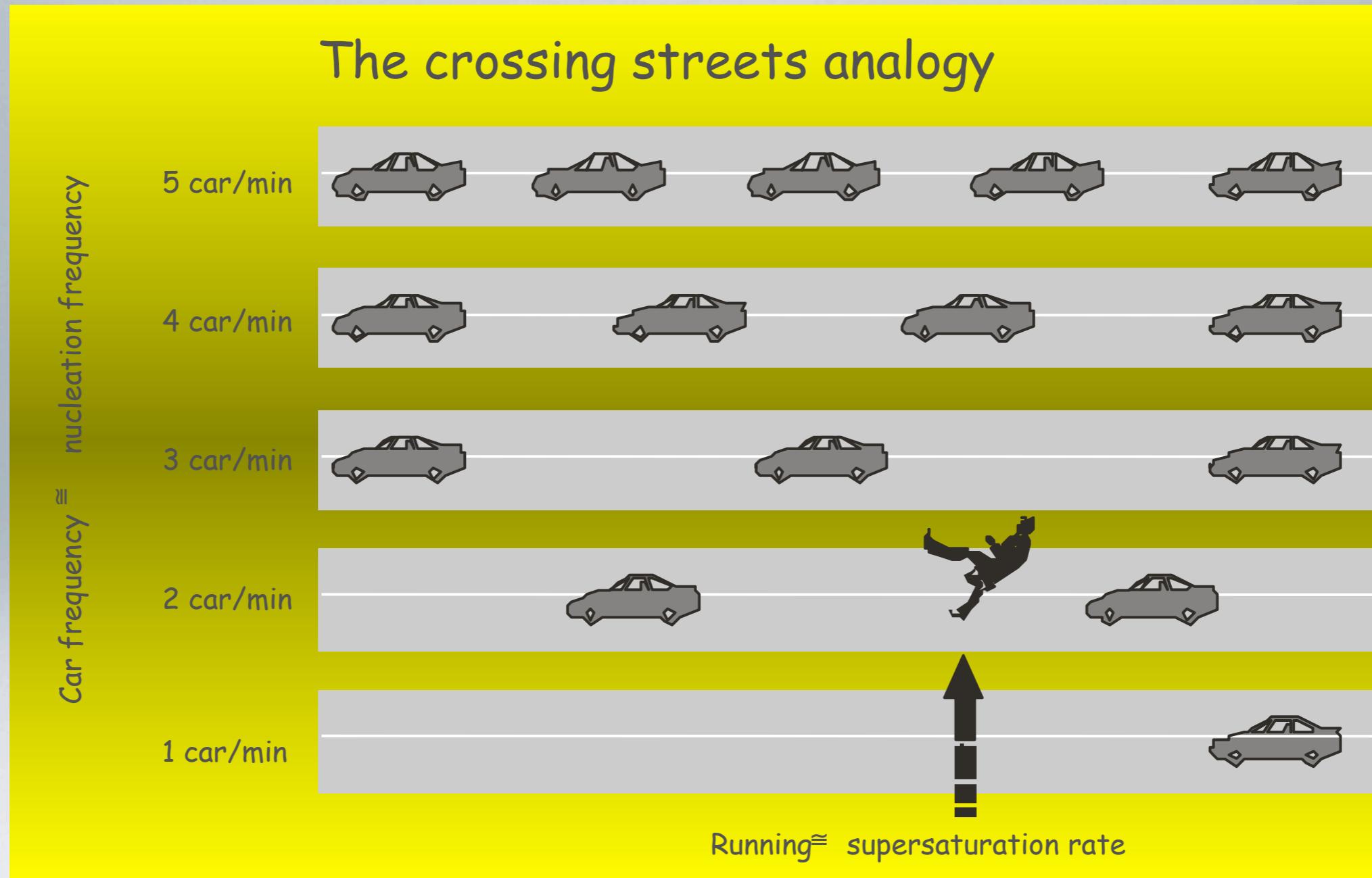
$$t = I/J$$

## Nucleation is a probabilistic phenomenon



J. M. García-Ruiz, Nucleation of protein crystals, Journal of Structural Biology, 142 (2003) 22-31

## Nucleation is a probabilistic phenomenon

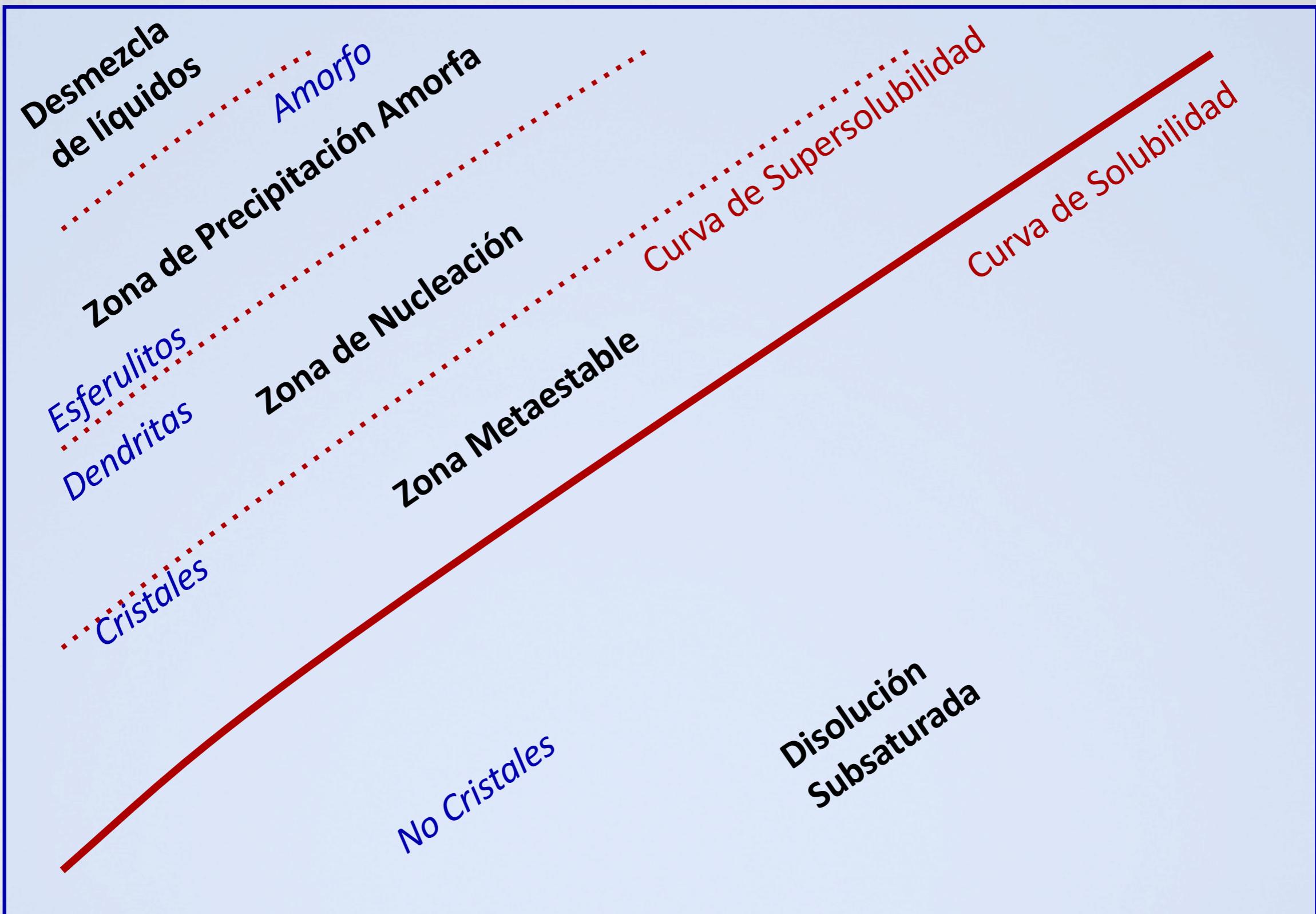


J. M. García-Ruiz, Nucleation of protein crystals, *Journal of Structural Biology*, 142 (2003) 22-31.

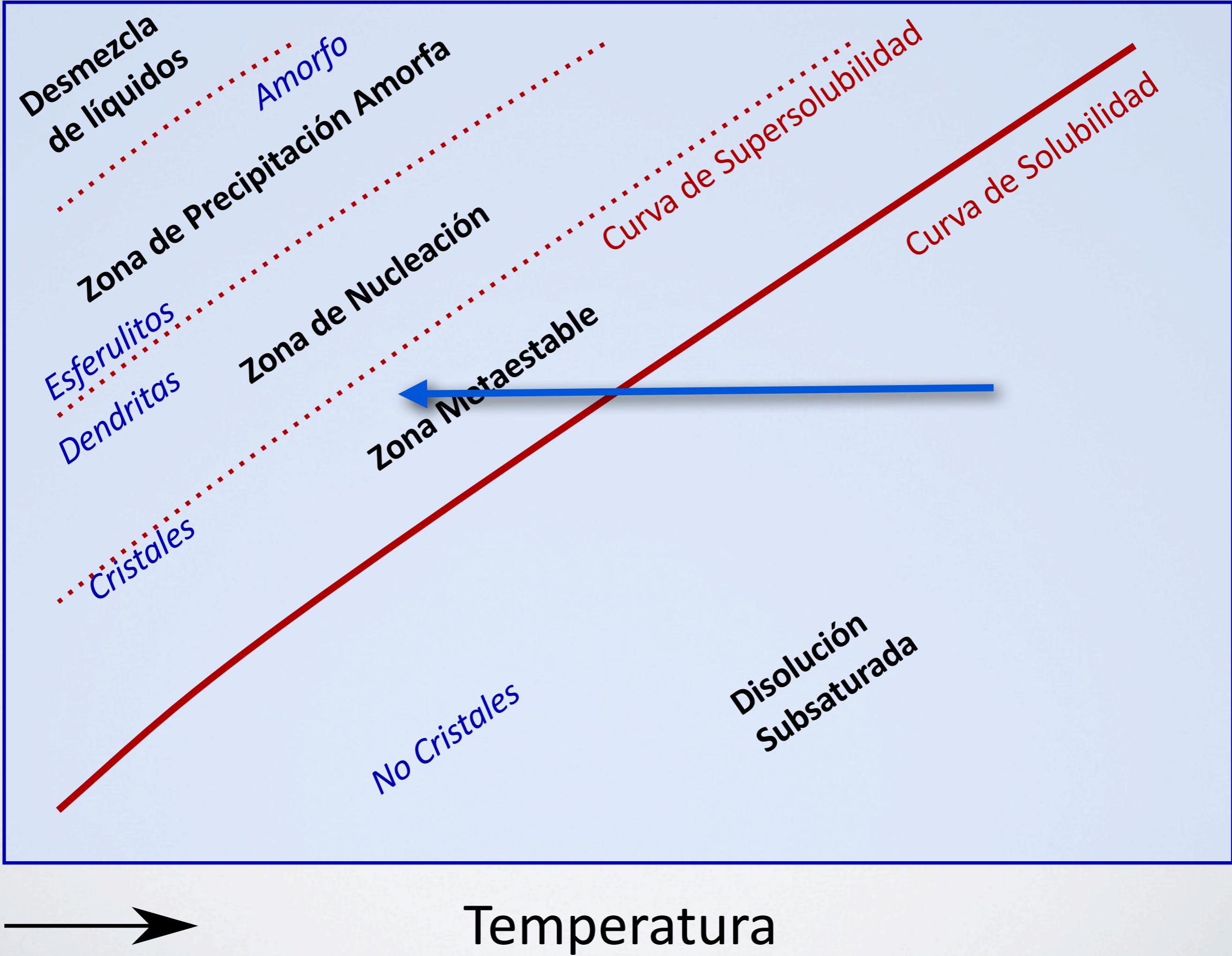
Concentración



Temperatura



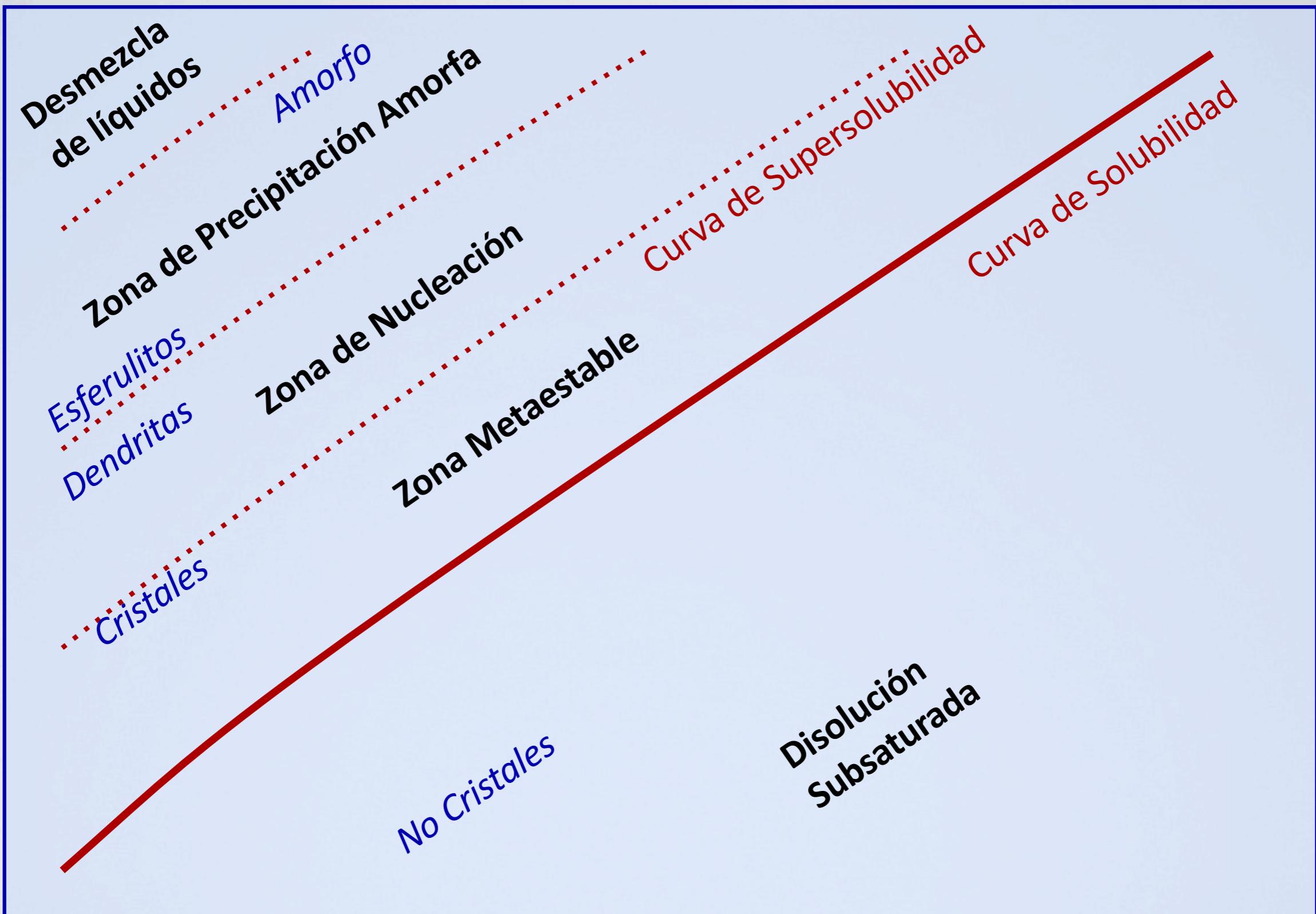
Concentración



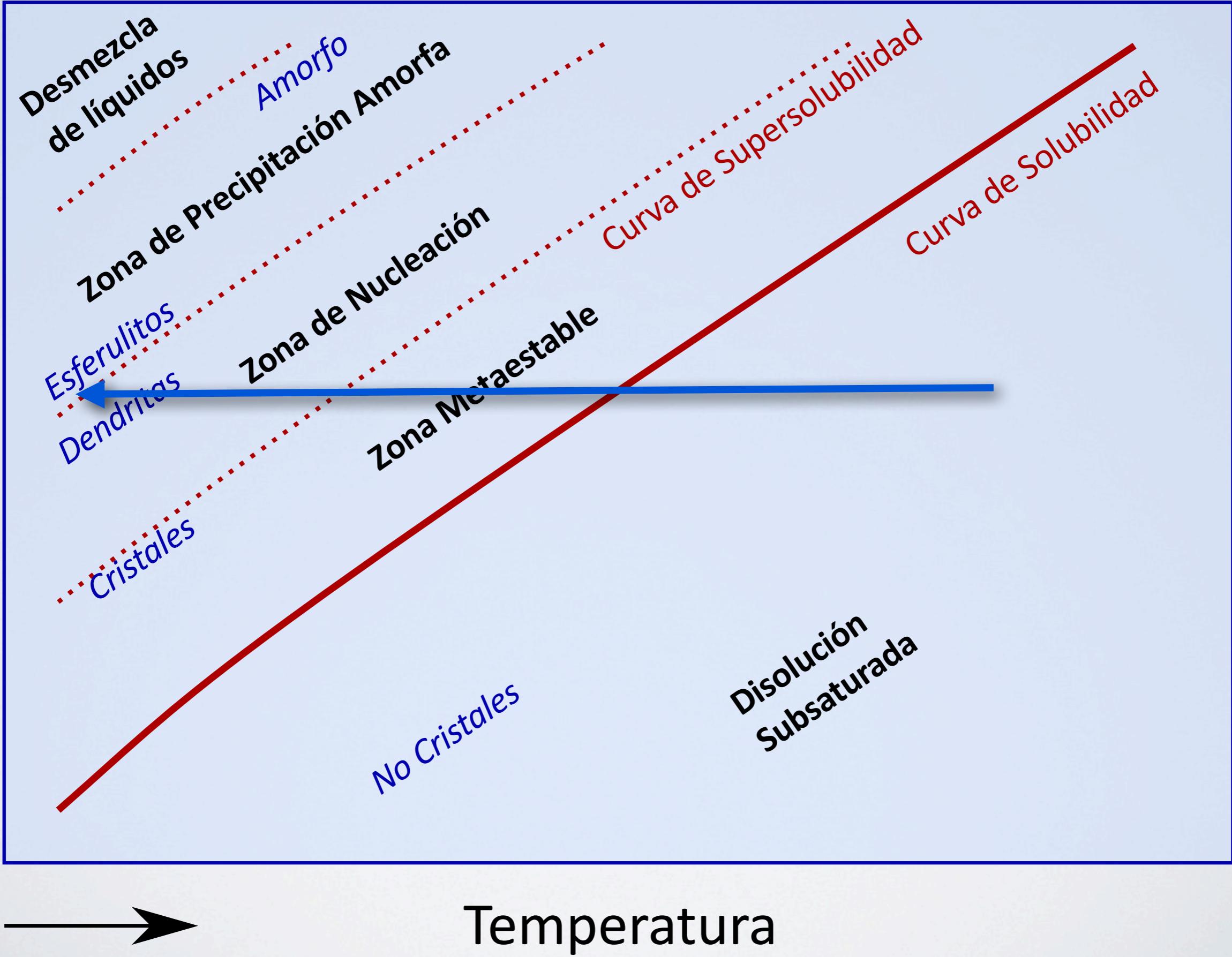
Concentración



Temperatura



Concentración



## **Homogeneous Nucleation**

Spontaneously in the bulk of pure solution

### Primary Nucleation

## **Heterogeneous Nucleation**

Induced by surfaces other than the forming crystals

### **Nucleation**

### Secondary Nucleation

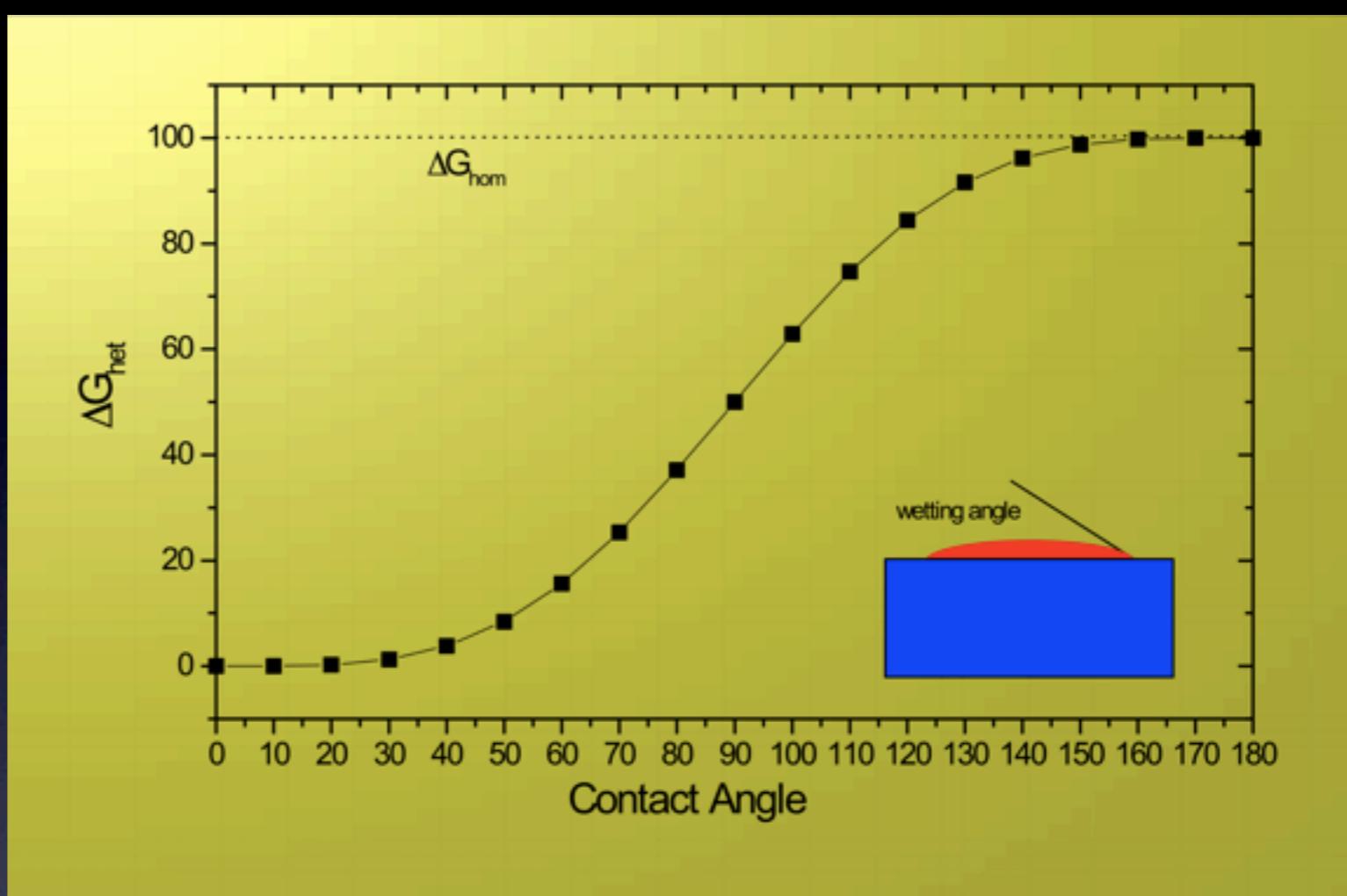
Induced by the forming crystals

**In most real systems, nucleation is heterogeneous**

# Heterogeneous nucleation

$$\Delta G_{\text{het}} = \Delta G_{\text{hom}} \left( \frac{1}{2} - \frac{3}{4} \cos \alpha + \frac{1}{4} \cos^3 \alpha \right)$$

Seeding:  
The crystal is used as substrate



If we plot the above expression for a given value of  $\Delta G_{\text{hom}}$  we found that if the nucleus wet the substrate ( $\alpha = 180^\circ$ ), then  $\Delta G_{\text{het}} = \Delta G_{\text{hom}}$ ; if  $\alpha = 90^\circ$ ,  $\Delta G_{\text{het}} = \frac{1}{2} \Delta G_{\text{hom}}$ . Smaller the value of  $\alpha$ , smaller the value of the activation energy for nucleation and it tends to zero for  $\alpha = 0^\circ$ . In general, any form of nucleation takes place by action of a solid surface composition different from that of the compound that crystallizes is called heterogeneous nucleation.

[http://www.dailymotion.com/video/xacdudc\\_homogeneous-nucleation\\_school](http://www.dailymotion.com/video/xacdudc_homogeneous-nucleation_school)

# Heterogeneous nucleation

$$\Delta G_{\text{het}} = \Delta G_{\text{hom}} \left( \frac{1}{2} - \frac{3}{4} \cos \alpha + \frac{1}{4} \cos^3 \alpha \right)$$

• The activation energy for homogeneous nucleation is given by

$\Delta G_{\text{hom}} = \frac{RT^2}{4} \ln \left( \frac{P_0}{P_c} \right)$

• The activation energy for heterogeneous nucleation is given by

$\Delta G_{\text{het}} = \Delta G_{\text{hom}} \left( \frac{1}{2} - \frac{3}{4} \cos \alpha + \frac{1}{4} \cos^3 \alpha \right)$

• The supersaturation required for homogeneous nucleation is

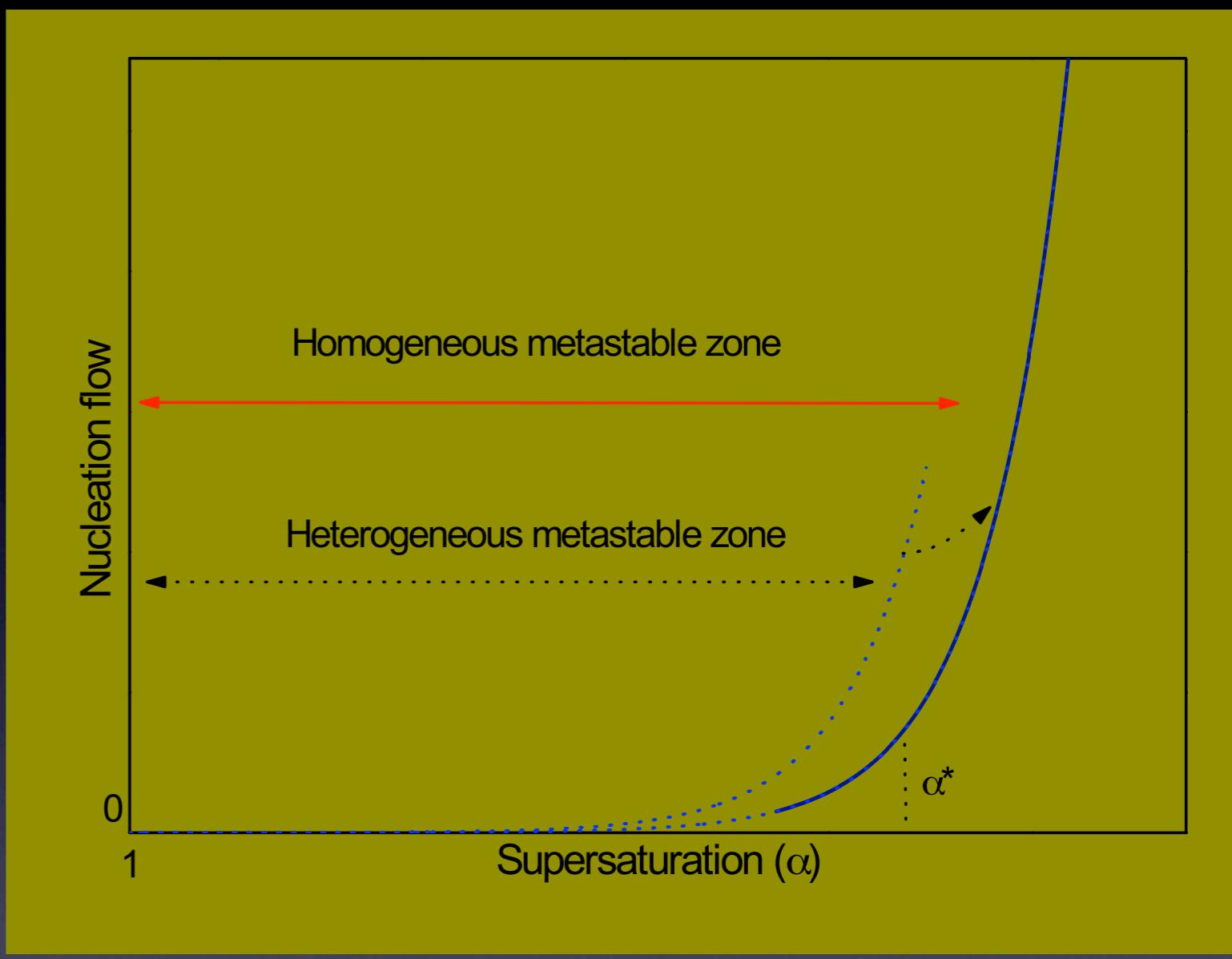
$\alpha_{\text{hom}} = \sqrt{\frac{P_c}{P_0}}$

• The supersaturation required for heterogeneous nucleation is

$\alpha_{\text{het}} = \sqrt{\frac{P_c}{P_0}} \left( \frac{1}{2} - \frac{3}{4} \cos \alpha + \frac{1}{4} \cos^3 \alpha \right)^{-1}$

**Seeding:**

The crystal is used as substrate



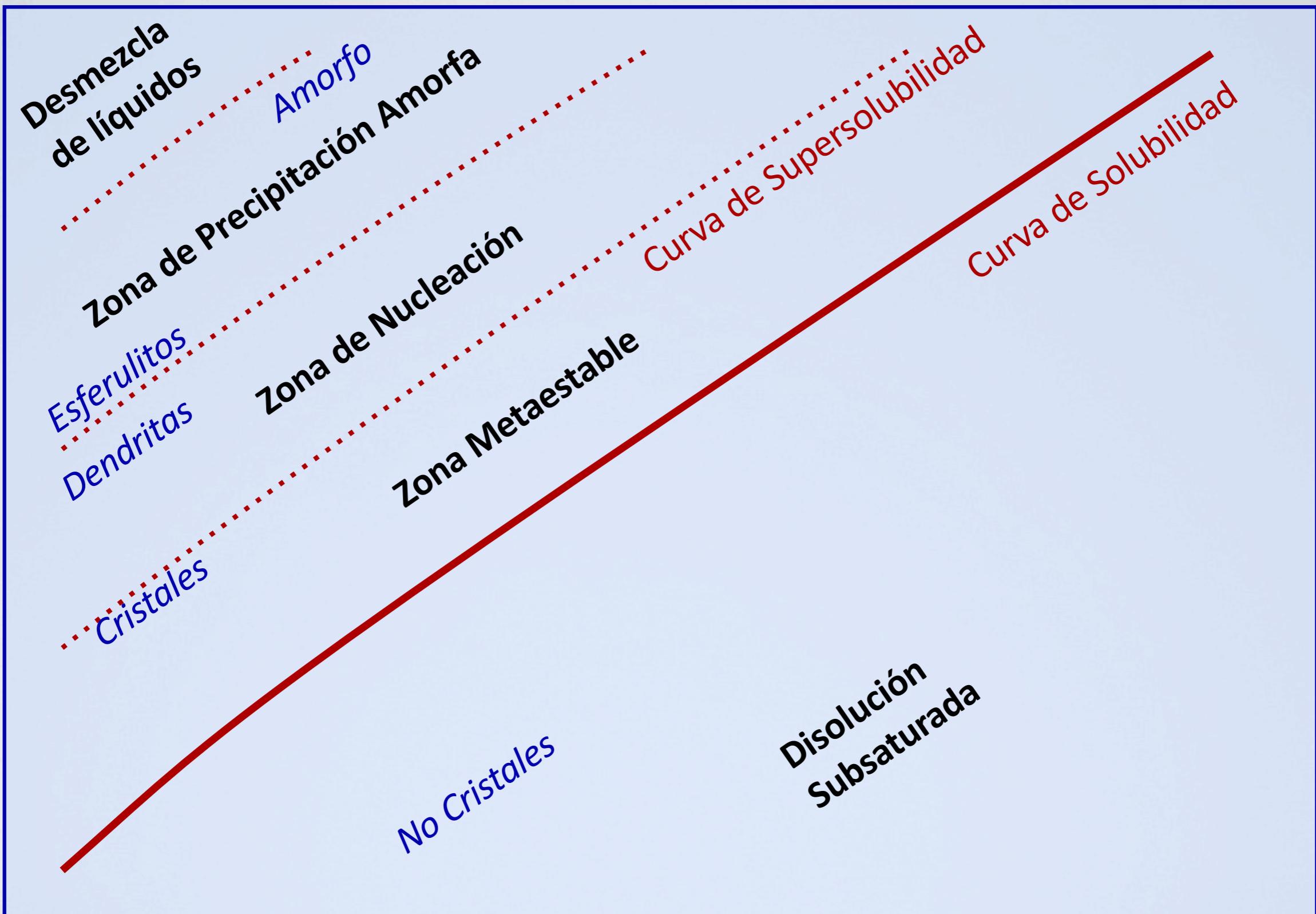
If we plot the above expression for a given value of  $\Delta G_{\text{hom}}$  we found that if the nucleus wet the substrate ( $\alpha = 180^\circ$ ), then  $\Delta G_{\text{het}} = \Delta G_{\text{hom}}$ ; if  $\alpha = 90^\circ$ ,  $\Delta G_{\text{het}} = \frac{1}{2} \Delta G_{\text{hom}}$ . Smaller the value of  $\alpha$ , smaller the value of the activation energy for nucleation and it tends to zero for  $\alpha = 0$ . In general, any form of nucleation takes place by action of a solid surface composition different from that of the compound that crystallize is called heterogeneous nucleation.

[http://www.dailymotion.com/video/xacduc\\_homogeneous-nucleation\\_school](http://www.dailymotion.com/video/xacduc_homogeneous-nucleation_school)

Concentración



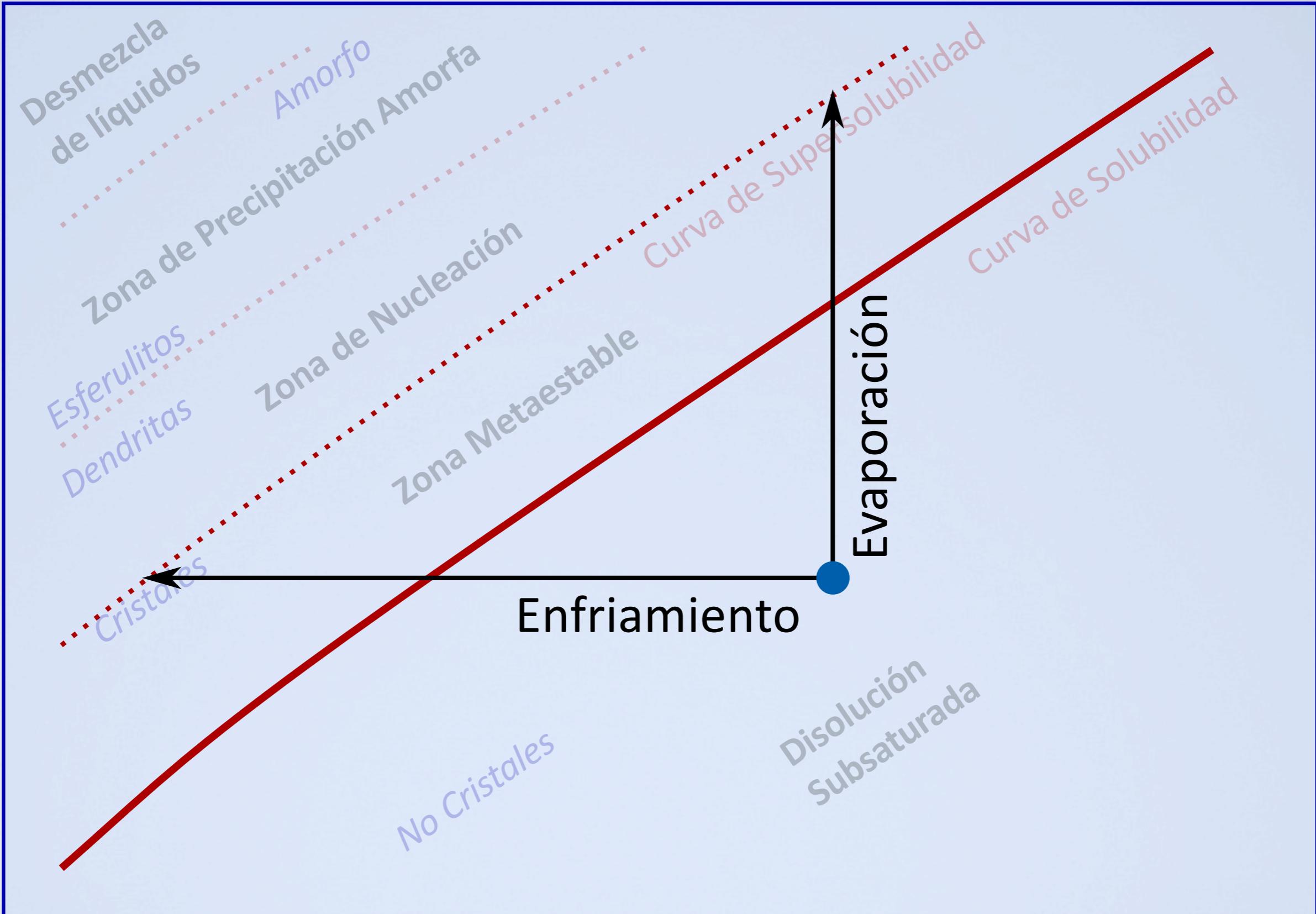
Temperatura



Concentración



Temperatura



Concentración



*Amorfo*

**Zona de Precipitación Amorfa**

*Esferulitos*

*Dendritas*

**Zona de Nucleación**

*Cristales*

**Curva de Supersolubilidad**

**Zona Metaestable**

**Curva de Solubilidad**

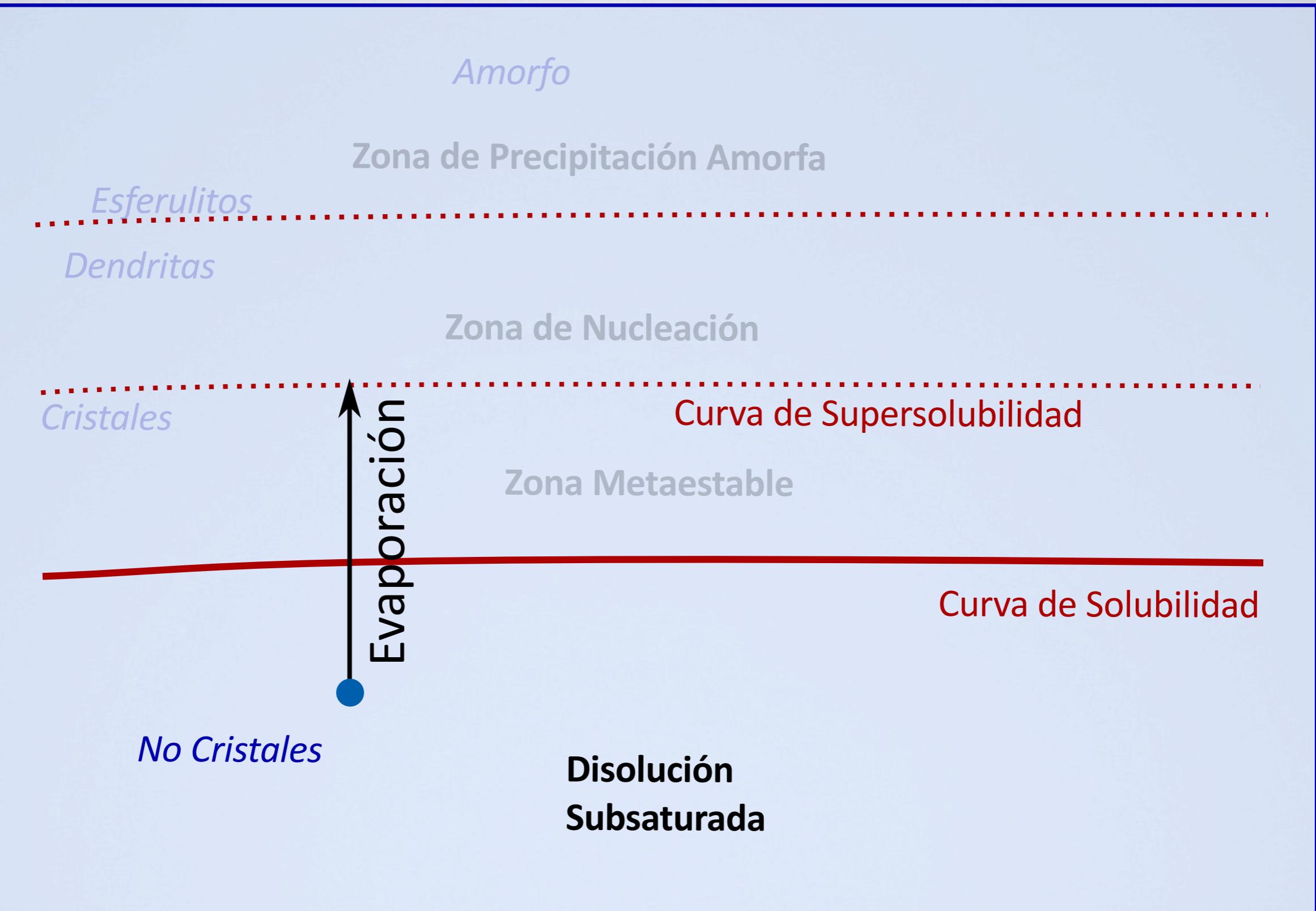
*No Cristales*

**Disolución  
Subsaturada**

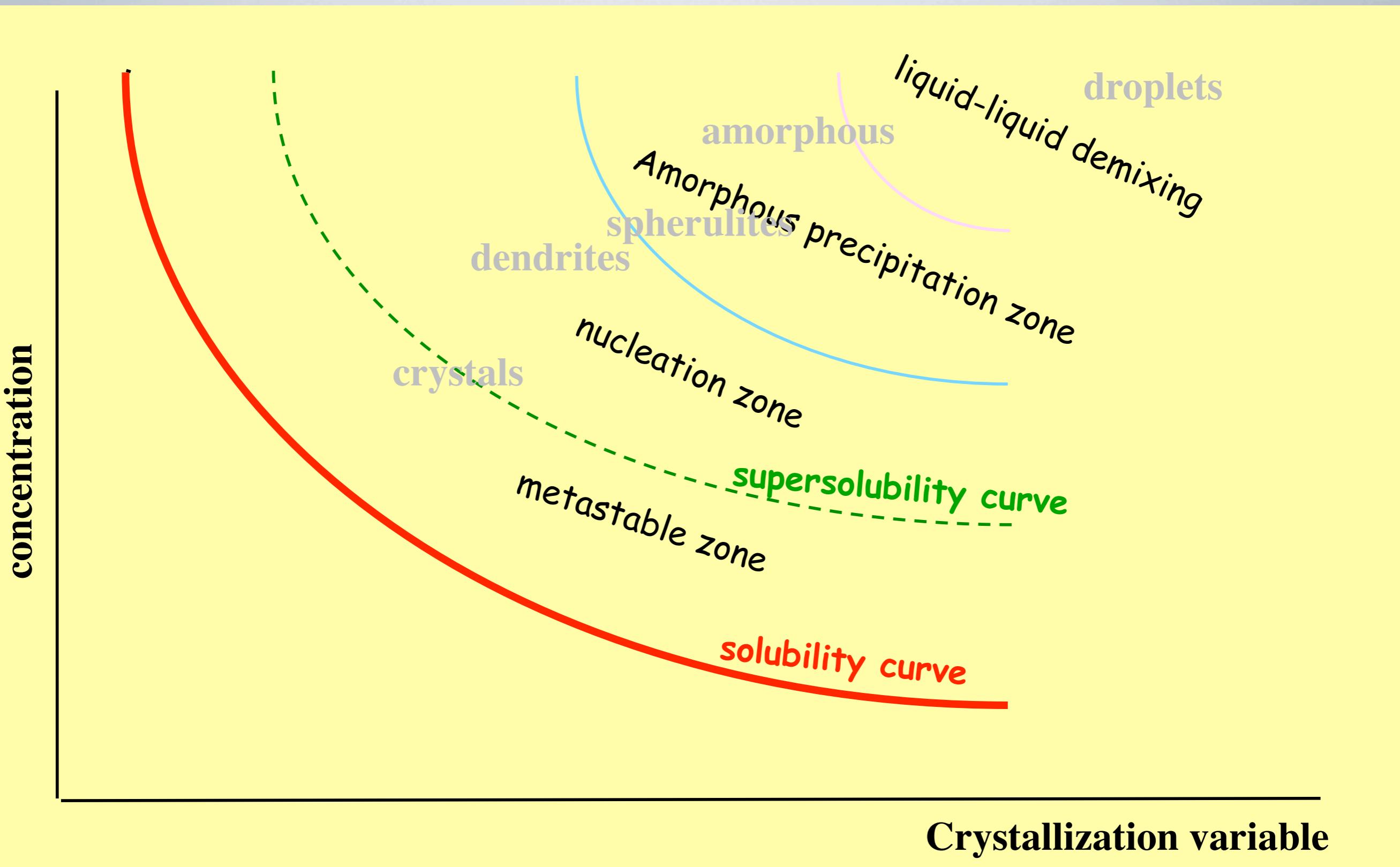


**Temperatura**

Concentración

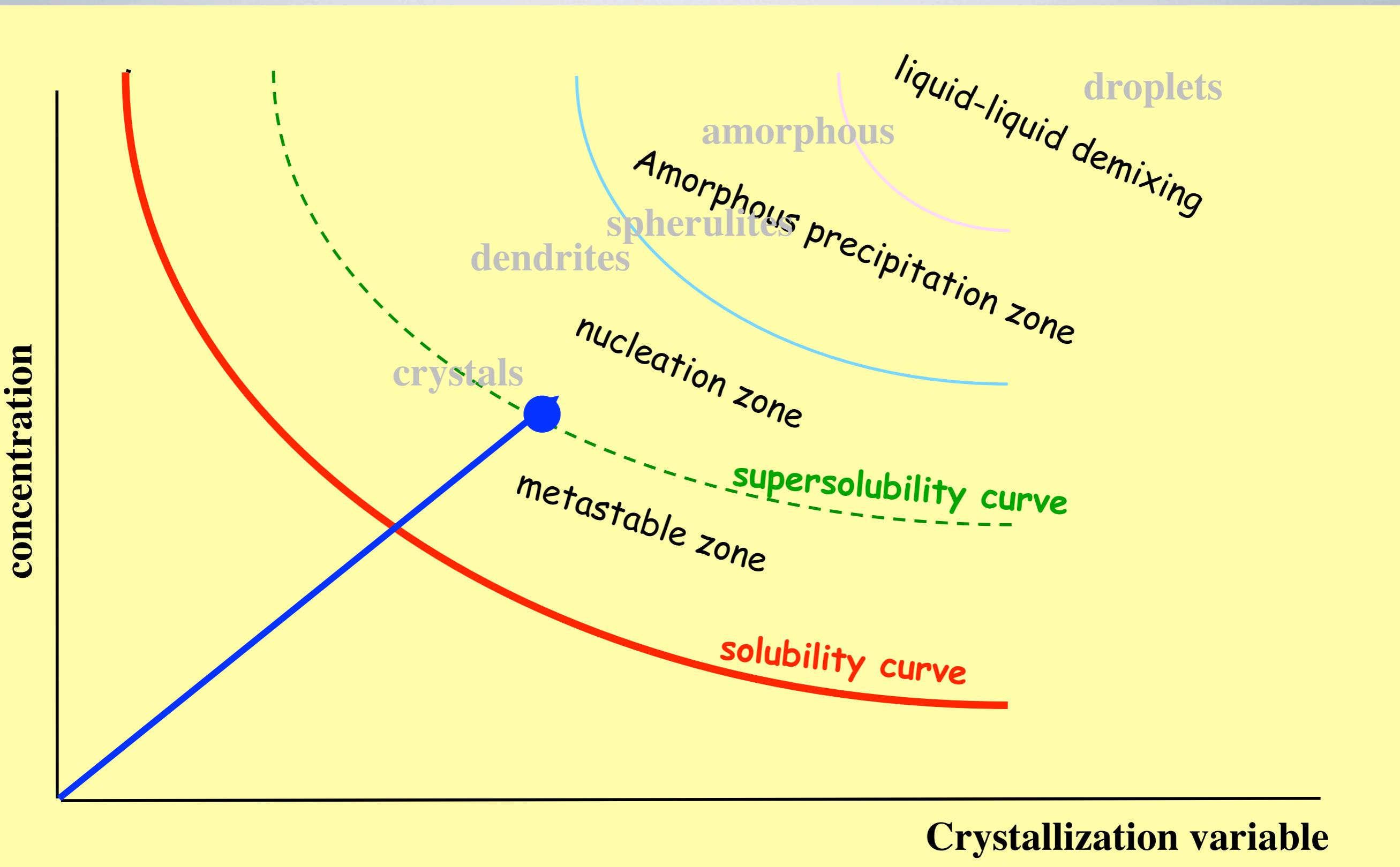


Temperatura



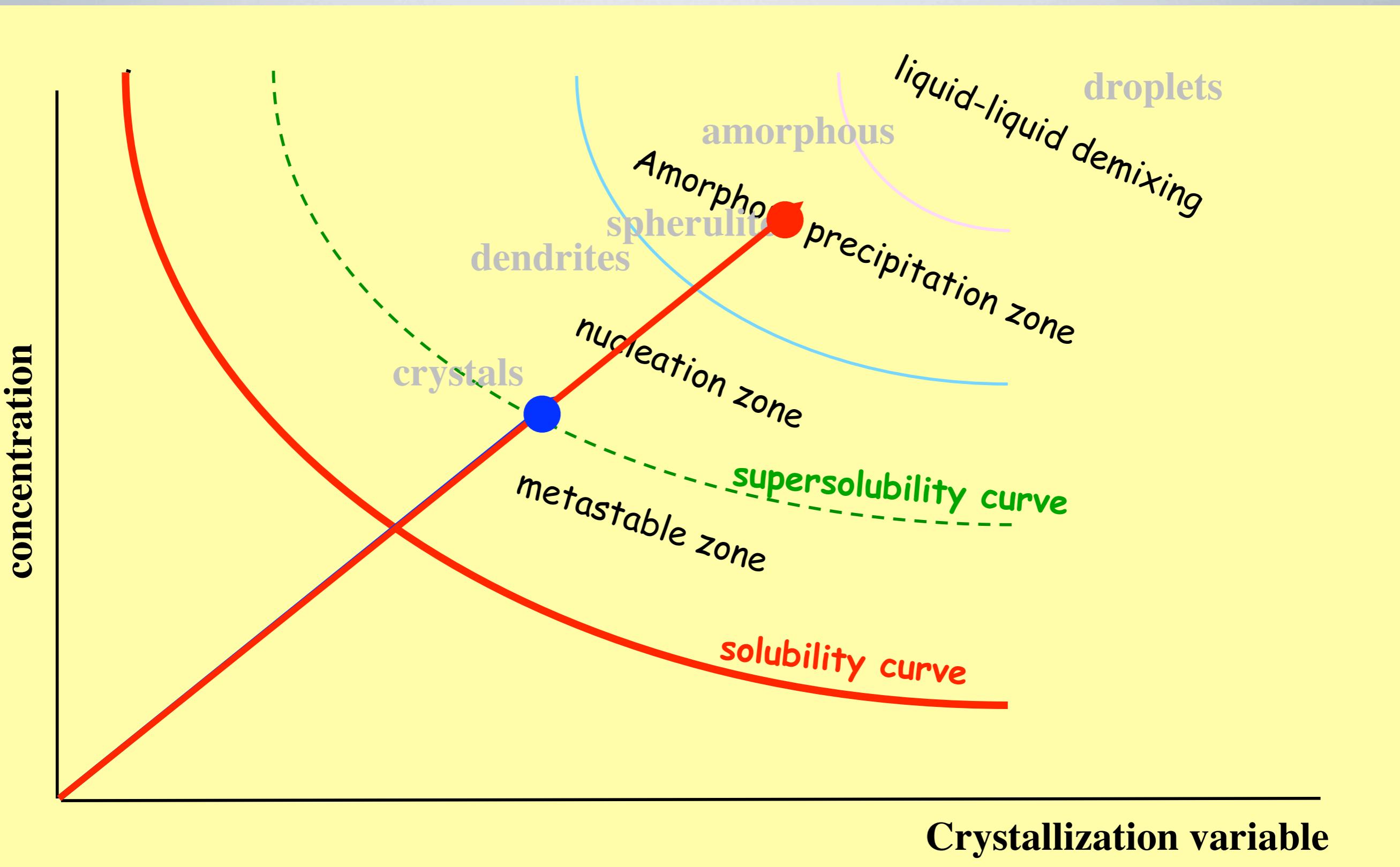
For the same path in the solubility diagram (i.e., for the same chemistry), the result of a crystallization experiment may vary drastically as a function of the rate of development of supersaturation

J. M. García-Ruiz, Nucleation of protein crystals, *Journal of Structural Biology*, 142 (2003) 22-31.



For the same path in the solubility diagram (i.e., for the same chemistry), the result of a crystallization experiment may vary drastically as a function of the rate of development of supersaturation

J. M. García-Ruiz, Nucleation of protein crystals, *Journal of Structural Biology*, 142 (2003) 22-31.



For the same path in the solubility diagram (i.e., for the same chemistry), the result of a crystallization experiment may vary drastically as a function of the rate of development of supersaturation

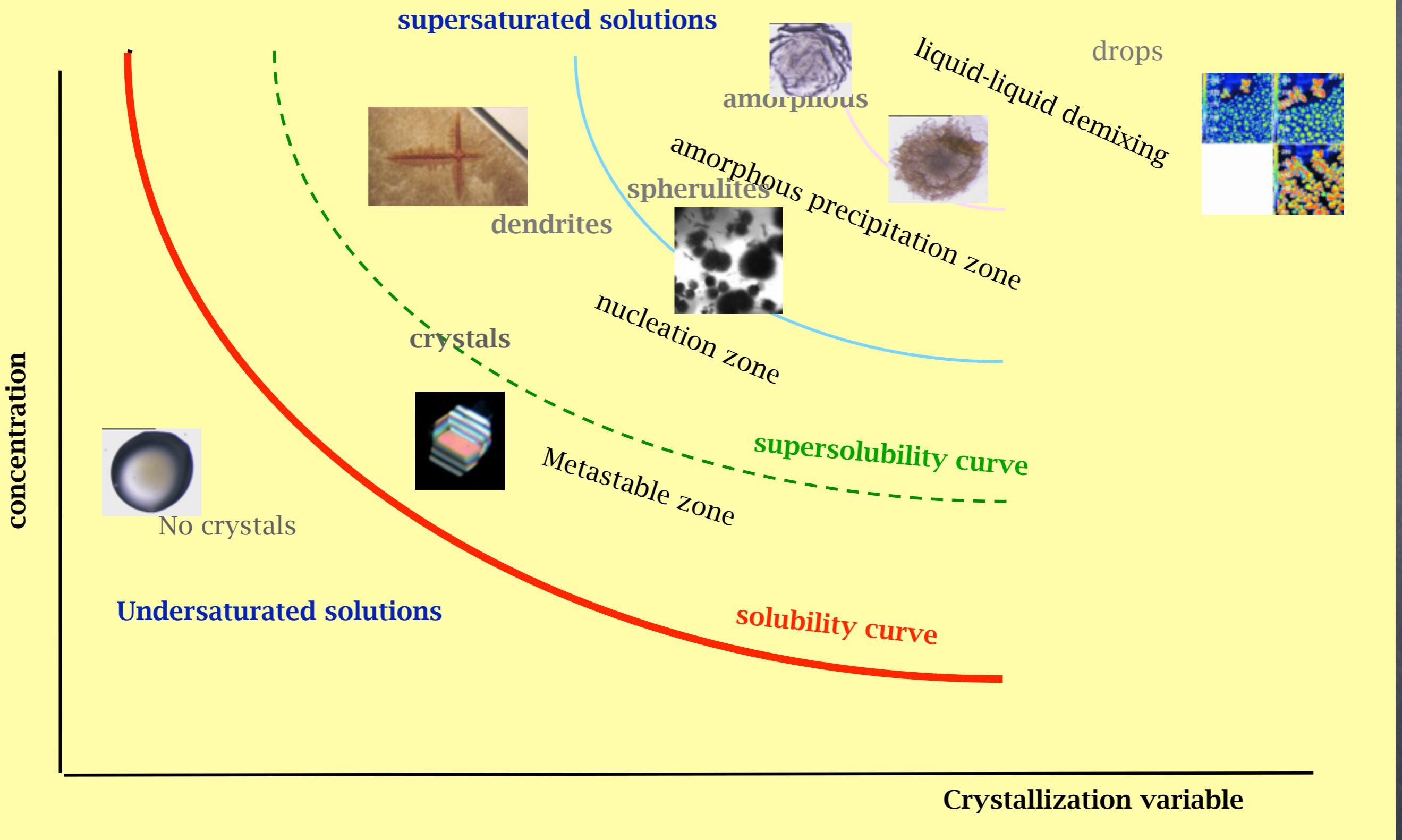
J. M. García-Ruiz, Nucleation of protein crystals, *Journal of Structural Biology*, 142 (2003) 22-31.

## supersaturated solutions

### Undersaturated solutions

Crystallization variable

This diagram represents the zones for the different states of the solute (dissolved, solid or liquid) for a range of solute concentration as a function of a crystallisation variable (for example, concentration of solubility reductor, pH, temperature, etc) all other variables being kept constant.



This diagram represents the zones for the different states of the solute (dissolved, solid or liquid) for a range of solute concentration as a function of a crystallisation variable (for example, concentration of solubility reductor, pH, temperature, etc) all other variables being kept constant.